Second JNT Biennial Conference, July 2022
Titles and Abstracts

Gregorio Baldi: Three finiteness theorems

I will present three seemingly unrelated finiteness results, and explain how the functional transcendence point of view relates them. One is about totally geodesic subvarieties in non-arithmetic ball quotients (joint with E. Ullmo), the second characterises the geometry of the Hodge locus and the third studies integral points of the moduli space of smooth hypersurfaces. The last two are joint work with E.Ullmo and B.Klingler.

Ashay Burungale: Zeta elements over imaginary quadratic fields

For an elliptic curve defined over the rationals, the talk plans to outline the existence of two-variable zeta element over an imaginary quadratic field and its applications to special cases of the BSD conjecture (joint with Chris Skinner and Ye Tian).

Francesco Calegari: Part I
Vessilin Dimitrov: Part II } The arithmetic of power series
Yunqing Tang: Part III

The unbounded denominators conjecture, first raised by Atkin and Swinnerton-Dyer in 1968, asserts that a modular form for a finite index subgroup of $\mathrm{SL}_2(\mathbb{Z})$ whose Fourier coefficients have bounded denominators must be a modular form for some congruence subgroup. The goal of this series of talks is to explain the main ingredients of the proof.

Laura DeMarco: Rigidity and uniformity in algebraic dynamics

The periodic orbits and their structure are fundamental features of a dynamical system. In an algebraic setting, where the system is defined by polynomials, we can use tools from algebraic or arithmetic geometry to study these orbits. Important special cases include endomorphisms of abelian varieties, for example in the uniform versions of the Mordell or Manin-Mumford Conjectures in the recent work of Dimitrov-Gao-Habegger, Kühne, Yuan and others, where the torsion points of the group coincide with the preperiodic points of an endomorphism. In this talk, I will describe some parallel questions and recent progress on more general families of complex and arithmetic dynamical systems.

Daniel Disegni: Theta cycles

I will survey an emerging theory of *'canonical'* algebraic cycles for motives enjoying a certain symmetry – for instance, some symmetric powers of elliptic curves. The construction is based on works of Kudla and Y. Liu on some (conjecturally modular) theta series valued in Chow groups of Shimura varieties.

These *'theta cycles'* seem as pleasing as Heegner points on elliptic curves:

(1) their nontriviality is detected by derivatives of complex or p-adic L-functions;

(2) if nontrivial, they generate the Selmer group of the motive. This supports analogues of the Birch and Swinnerton-Dyer conjecture. (Works by C. Li-Liu, D.-Liu, D.)

Ziyang Gao: Torsion points in families of abelian varieties

Given an abelian scheme defined over $\overline{\mathbb{Q}}$ and an irreducible subvariety X which dominates the base, the Relative Manin-Mumford Conjecture (proposed by Zannier) predicts how torsion points in closed fibers lie on X. The conjecture says that if such torsion points are Zariski dense in X, then the dimension of X is at least the relative dimension of the abelian scheme, unless X is contained in a proper subgroup scheme. In this talk, I will present a proof of this conjecture. As a consequence this gives a new proof of the Uniform Manin-Mumford Conjecture for curves (recently proved by Kühne) without using equidistribution. This is joint work with Philipp Habegger.

Mahesh Kakde: On Brumer–Stark conjecture and refinements

The talk will start with a statement of the Brumer–Stark conjecture. I will then give a formulation of a strong refinement of the Brumer–Stark conjecture using Ritter–Weiss module. I will then sketch a proof of this conjecture. This is all joint work with Samit Dasgupta.

Lars Kühne: The Relative Bogomolov Conjecture for Fibered Products of Elliptic Families

I will talk about the deduction of the Bogomolov conjecture for non-degenerate subvarieties in fibered products of elliptic families from equidistribution in families of abelian varieties. This generalizes results of DeMarco and Mavraki and improves certain results of Manin-Mumford type proven by Masser and Zannier (or most recently: Gao and Habegger) to results of Bogomolov type.

Brian Lawrence: Sparsity of Integral Points on Moduli Spaces of Varieties

Interesting moduli spaces don't have many integral points. More precisely, if X is a variety over a number field, admitting a variation of Hodge structure whose associate period map is injective, then the number of S-integral points on X of height at most H grows more slowly than $H\varepsilon$, for any positive ε. This is a sort of weak generalization of the Shafarevich conjecture; it is a consequence of a point-counting theorem of Broberg, and the largeness of the fundamental group of X. Joint with Ellenberg and Venkatesh.

Chao Li: From sum of two squares to arithmetic Siegel-Weil formulas

We begin with the classical sum of two squares problem and put it in the modern perspective of the Siegel–Weil formula. After illustrating Kudla's influential program on geometric and arithmetic generalizations, we survey recent development on arithmetic Siegel–Weil formulas for higher dimensional Shimura varieties and applications.

Paul Nelson: Bounds for standard L-functions

We consider the standard L-function attached to a cuspidal automorphic representation of a general linear group. We present a proof of a subconvex bound in the t-aspect. More generally, we address the spectral aspect in the case of uniform parameter growth.

These results are the subject of third paper linked below, building on the first two.

https://arxiv.org/abs/1805.07750
https://arxiv.org/abs/2012.02187
https://arxiv.org/abs/2109.15230

James Newton: Modularity of elliptic curves over CM fields

Since the seminal works of Wiles and Taylor–Wiles, robust methods were developed to prove the modularity of *'polarised'* Galois representations. These include, for example, those coming from elliptic curves defined over totally real number fields. Over the last 10 years, new developments in the Taylor-Wiles method (Calegari, Geraghty) and the geometry of Shimura varieties (Caraiani, Scholze) have broadened the scope of these methods. One application is the recent work of Allen, Khare and Thorne, who prove modularity of a positive proportion of elliptic curves defined over a fixed imaginary quadratic field. I'll review some of these developments and work in progress with Caraiani which has further applications to modularity of elliptic curves over imaginary quadratic fields.

Jonathan Pila: Ax–Schanuel and exceptional integrability

In joint work with Jacob Tsimerman we study when the primitive of a given algebraic function can be constructed using primitives from some given finite set of algebraic functions, their inverses, algebraic functions, and composition. When the given finite set is just $\{1/x\}$ this is the classical problem of *"elementary integrability."* We establish some results, including a decision procedure for this problem.

Raphael Von Kanel: Integral points on coarse Hilbert moduli schemes

In this talk, I will present explicit bounds for the height and the number of integral points on coarse Hilbert moduli schemes outside the branch locus. Furthermore, I will illustrate the results with examples given by certain classical surfaces and I will explain the strategy of proof which combines the method of Faltings (Arakelov, Parsin, Szpiro) with modularity and Masser-Wuestholz isogeny estimates. This is joint work with Arno Kret.

Sarah Zerbes: Euler systems and the Birch–Swinnerton-Dyer conjecture for abelian surfaces

Euler systems are one of the most powerful tools for proving cases of the Bloch–Kato conjecture, and other related problems such as the Birch and Swinnerton-Dyer conjecture.

I will recall a series of recent works (variously joint with Loeffler, Pilloni, Skinner) giving rise to an Euler system in the cohomology of Shimura varieties for GSp(4), and an explicit reciprocity law relating the Euler system to values of L-functions. I will then explain recent work with Loeffler, where we use this Euler system to prove new cases of the BSD conjecture for modular abelian surfaces over $\mathbb{Q}$, and for modular elliptic curves over imaginary quadratic fields.